Reasoning Under Uncertainty: Belief Updating in Simple Bayesian Framework
An Example

Given: Prior probability that this city is location of next terrorist attack:

Los Angeles	*Cincinnati*	*Columbus*	*Dayton*	*NYC*	*Buffalo*	*Detroit*
0.20	0.05	0.10	0.15	0.20	0.20	0.10

A. Certain evidence: Simple conditioning $P_{new}(A) = P(A|E) = P(A \cap E)/P(E)$

Example

Evidence: "The attack will occur in Ohio"

So E= {Cincinnati, Columbus, Dayton}

After update:

Los Angeles	*Cincinnati*	*Columbus*	*Dayton*	*NYC*	*Buffalo*	*Detroit*
0	1/6	2/6	3/6	0	0	0

E.g. P_{new}({Columbus})

= P({Columbus} | {Cincinnati, Columbus, Dayton})

= P({Columbus}) / P({Cincinnati, Columbus, Dayton})

= 0.10 / 0.30 = 2/6

B. Uncertain evidence: Jeffrey's Rule $P_{new}(A) = \sum_i P(A \mid E_i)\, P_{new}(E_i)$

Example

Evidence: "The probability states will be attacked: CA : 0.10 OH: 0.60 NY: 0.20 MI: 0.10"

[Cf. *prior* probabilities of same events: CA : 0.20 OH: 0.30 NY: 0.40 MI: 0.10]

So E_i = event [world] where state i is attacked

Originally: P(OH) = P({Cincinnati, Columbus, Dayton }) = 0.30, etc.

Now given: P_{new}(OH) = P_{new}({Cincinnati, Columbus, Dayton }) = 0.60, etc.

After update:

Los Angeles	*Cincinnati*	*Columbus*	*Dayton*	*NYC*	*Buffalo*	*Detroit*
0.10	0.10	0.20	0.30	0.10	0.10	0.10

Special case of singleton events:

$P_{new}(\{x\}) = P(\{x\}|E_i)\, P_{new}(E_i)$ where E is the event containing x.

E.g. P_{new}({Columbus})

= P({Columbus}|OH) P_{new}(OH)

= (2/6) 0.60 = 0.20

Note:

Like Bayes Theorem, Jeffrey's Rule is a nontrivial heuristic interpretation of a trivial theorem:

$$P(A) = \sum_i P(A \mid E_i)\, P(E_i) \quad \text{where } \{E_i\} \text{ is a partition of the space.}$$

The condition $P_{new}(A|E_i) = P(A|E_i)$ is required for Jeffrey's theorem to be "correct." That is, the imagined P_{new} must satisfy this condition (which in reality we might never be able to know).